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REPLY TO COMMENTARY

Again on the optimal rotation period of renewable resources: wrong objections to a right objective

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Abstract
An article coming from my pen has met with opposition. If the critics are to be believed, I put forth an erroneous conception on how to calculate the present value of a forest mill's total assets, including soil and standing timber. This reproach, however, is untenable. In fact, the economically correct solution brings to light that the principle of sustainable maximum yield is productively efficient and at the same time most lucrative.

Keywords Renewable resources · Optimal rotation period · Faustmann formula · Maximum sustainable yield

JEL Classification Q23 · D92

Profit maximization in silviculture

At the outset of his often quoted article from 1849, Martin Faustmann posed the following problem: "…what is the net annual yield which *bare* forest land can provide in perpetuity?".[1] The answer to this question is needed to calculate the pecuniary loss in an insurance case, e. g. when fallow forest ground was destroyed by "fire, insects, man, etc".[2] In Faustmann's view, the money amount of the damage—and thus the volume of compensation—should be equal to the hypothetical earning power of the area, i.e. the indemnity payment matches the discounted forgone profits in future.

In order to develop the best harvesting strategy, established forest economics usually also takes an owner of untilled ground as a starting point. Furthermore, the person allegedly looks for the rotation that would make the naked soil most expensive. However, this is not the standard challenge which forest management faces in everyday business. Typically, entrepreneurs in silviculture strive to maximize the worth of their *total* assets, comprising land as well as stumpage.

In a recently published article,[3] I criticize the dominant approach as too narrow since it refers only to the land value. In contrast, I expounded the expedient behaviour to realize the broader defined objective including the tree population. These deliberations have been called into question. Yet, prior to the evaluation of the criticisms, a brief sketch of my main argument appears appropriate.

Following Faustmann, an endless chain of notional projects carried out on one hectare is considered: An infinite number of harvesting and reforestation processes follow an initial investment of the (constant) planting costs (L). Each cohort grows a designated time span (T) which Faustmann treated as given. From the yield $f(T)$ (net of felling expenditures), the seedlings are bought for the next round. In the continuous case with the (invariable) interest rate (i), the imputed present value of the uncultivated soil (PV_S) amounts to:

$$PV_S(T) = -L + (f(T) - L)\, e^{-iT} + (f(T) - L)\, e^{-2iT} + \cdots \quad (1)$$

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[1] Faustmann (1968, p. 28, italics added). The passage reads in German: "Welches ist der reine Geldertrag, den ein jetzt holzleerer Waldboden immerwährend in jährlich gleicher Größe liefert?" Faustmann (1849, p. 442).

[2] Faustmann (1968, p. 27). In the original, there is talk of "Waldzerstörungen durch Feuer, Insekten, Menschen …" Faustmann (1849, p. 441).

[3] Helmedag (2018).

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Rearranging this series leads to:

$$\mathrm{PV}_S(T) = \left(f(T)\,e^{-iT} - L\right) + \left(f(T)e^{-iT} - L\right)\,e^{-iT} + \left(f(T)e^{-iT} - L\right)\,e^{-2iT} + \cdots \quad (2)$$

Applying the rule for infinite sums entails:

$$\mathrm{PV}_S(T) = \frac{f(T)e^{-iT} - L}{1 - e^{-iT}} = \frac{f(T) - Le^{iT}}{e^{iT} - 1} \geq 0 \quad \text{for } f(T) - Le^{iT} \geq 0 \quad (3)$$

Thus, the damage claim reaches the total of the "land expectation value" where all parameters—including the production cycle T—are fixed. From another perspective, this sum constitutes the equilibrium price for the unused plot when arbitrage and speculation possibilities are excluded.

Running a timber company in capitalism, however, does not only consist in calculating the adequate compensation for the destruction or selling of an uncultivated field. In daily business, areas *are* tilled and the forester wants to know how long a tree should mature. Normally, the enterprises' property includes the terrain *plus* the stumpage upon it. Contrary to his successors, Faustmann had appreciated this aspect: "However, if land carrying wood is sold, and if the vendor therefore has to sell the present stand at its current market value, then the buyer should compensate the vendor for the loss which results, as well as the land value".[4] Of course, the values of soil and timber depend on the rotation period T.

The general formula to calculate the company's present value of total assets, i.e. the afforested ground, at a date $t \leq T$ ($\mathrm{PV}_A(t,T)$) reads:

$$\mathrm{PV}_A(t,T) = \left(\mathrm{PV}_S(T) + f(T)\right)e^{-i(T-t)} \quad \text{with } 0 \leq t \leq T \quad (4)$$

Instead of an even-aged crop growing on the hectare under consideration, harvesting can also be organized continuously. Then, the area is divided into T parcels each planted with trees of a certain vintage ranging from the beginning to the end of their lifetime. Inserting Eq. (3) in formula (4), integrating over t and finally dividing by T gives the average value of the timber company's total assets $\left(\text{Ø}\mathrm{PV}_A(T)\right)$.[5] At a first glance, the ensuing function looks quite complex but it can be simplified to a pretty compact term:

$$\text{Ø}\mathrm{PV}_A(T) = \frac{\int_0^T \left(f(T) + \frac{f(T)e^{-iT} - L}{1 - e^{-iT}}\right) e^{-i(T-t)}\mathrm{d}t}{T} = \frac{f(T) - L}{iT} \quad (5)$$

In a "synchronized", "normal" or a "fully regulated" production, the same *sustainable yield* (SY) is reaped annually:

$$\mathrm{SY}(T) = \frac{f(T) - L}{T} \quad (6)$$

The present value of these recurrent incomes up to infinity ($\mathrm{PV}_{\mathrm{SY}(T)}$) converges to:

$$\mathrm{PV}_{\mathrm{SY}(T)} = \int_0^\infty \left(\frac{f(T) - L}{T}\right) e^{-it}\mathrm{d}t = \frac{f(T) - L}{iT} \quad (7)$$

Obviously, expressions (5) and (7) coincide. Thus, formula (6), sometimes named "forest rent", indeed turns out to be the appropriate stepping stone for optimizing timber output. Consequently, the principle of *maximum* sustainable yield alias the instruction of Joseph II informs about the superior rotation period in forestry (T*):

$$f'(T^*) = \frac{f(T^*) - L}{T^*} = i \cdot \text{Ø}\mathrm{PV}_A(T^*) \quad (8)$$

Alternatively, a graphical method can be deployed to obtain T*.[6] In contrast to the Faustmann condition, the best practice to manage renewable resources turns out to be independent of the interest rate. This outcome is not only important for a timber mill's most profitable operation, but also from the vantage point of environmental economics.

False assumptions, forged arguments and fallacious advices

Interestingly enough, the literature abounds in recommendations when to cut a tree.[7] The fewest of these precepts are simply (formally) wrong but suit diverse fields of application; i.e. different answers to different questions are given. Thus, first of all, the decision maker has to be clear about the *economic* goal (s)he pursues. For instance, if only a single plantation is considered, the application of the so-called Jevons–Fisher rule appears worthwhile. Yet, in capitalism enterprises typically want to rake in as much profit as possible as long as possible. Hence, "… the management of a timber company strives after the highest permanent gain per period which entails the greatest present value of the firm's

[4] Faustmann (1968, p. 32). In German: "Wird aber ein mit Holz bebauter Boden verkauft, und muß der Verkäufer daher den gegenwärtigen Holzbestand nach seinem jetzigen Verbrauchswerthe versilbern; so hat der Käufer, außer dem Bodenwerthe, noch den durch Letzteres entstehenden Verlust dem Verkäufer zu ersetzen."Faustmann (1849, p. 444).

[5] Intermittent and sustaining processes generate the same mean present value of the cultivated hectare irrespective of how long trees grow. See Helmedag (2018, p. 304).

[6] See Helmedag (2018, Fig. 1, p. 305).

[7] See Helmedag (2008).

assets".[8] Against this characterization neither Chang et al. (2019) nor Knoke et al. (2019) raised an objection. Therefore, the onus of proof that my analysis is erroneous rests with the critics. Did they champion a superior solution to the posed problem? No.

Let us begin with the protest advanced by Chang and his co-authors: "… Helmedag asserts that when the forest is fully regulated … the forest rent model is the correct model in choosing the optimal rotation age … we will first prove both mathematically and then demonstrate empirically that such a claim is patently false".[9] This account, however, reverses my way of proceeding. The very title of my paper indicates my aspiration: *reconciling* the Faustmann formula with the principle of maximum sustainable yield. Consequently, I based my research on Faustmann's actuarial calculation of the soil's present value, subsequently augmented by incorporating the worth of the stand. As a *result* of my enquiry, the forest rent approach proved to be proper.

In order to substantiate their reproach, Chang et al. (2019) start with introducing a new variable: "Following Helmedag's notation, given a fully regulated forest of A acres …"[10] Yet, I always referred to a metric measure, namely one hectare. Regardless, without loss of generality, in the equations advanced by Chang et al. (2019), the number of acres (A) can be set to unity.[11] The authors then launch a "net present value of the fully regulated forest (NPVF)".[12] Using the above symbols, their reckoning boils down to the following *definition*:

$$\mathrm{NPVF} = \emptyset\mathrm{PV}_A(T) - \left(\emptyset\mathrm{PV}_A(T) - \mathrm{PV}_S(T)\right) = \mathrm{PV}_S(T) \qquad (9)$$

The amount in brackets, i.e. the money equivalent of the stand, is detracted from the present value of soil and stumpage in order to get *by construction* only the naked land's worth. However, such reasoning leads astray. Under the prevailing decision circumstances, it is delusive to concoct a difference between a gross and a net present value of an existing forest enterprise. The authors justify their procedure with the claim that in the formula for the timber mill's assets (5) "… there is no accounting of the cost of acquiring such a fully regulated forest. To engender such a revenue stream, money must be spent to secure the fully regulated forest".[13]

Manifestly, Chang et al. (2019) misconstrue the present value of an already operating company. In a world of perfect foresight and no risk, the principal amount consists in adding up discounted future profits, i.e. upcoming periodical revenues minus costs, without deducting any sunk outlays whatsoever. The trees on the ground grow from incipience to maturity and have not been bought (without the carrying ground?) for a price equal to the present value of standing timber. But if nevertheless this sum is subtracted, why not the worth of the plot $\mathrm{PV}_S(T)$, too? In that case, the "real" net present value à la Chang et al. (2019) completely vanishes. Upon closer examination, this conclusion is less surprising as it may appear at first sight. In an equilibrium where perfect certainty excludes any speculation possibilities, an investor never can pick up a bargain since all capital goods are traded for their present value. Consequently, for *every* rotation period T the net yield of an even-aged forest just suffices to refund the accumulated interest on the expenditures for the uncultivated soil $\mathrm{PV}_S(T)$ plus planting costs L:

$$\begin{aligned}\left(\mathrm{PV}_S(T) + L\right)\left(e^{iT} - 1\right) &= \left(\frac{f(T)e^{-iT} - L}{1 - e^{-iT}} + L\right)\left(e^{iT} - 1\right) \\ &= f(T) - L\end{aligned} \qquad (10)$$

Notwithstanding the above, an owner of forest land wants to know when it is most profitable to cut a tree. Since the proclaimed "mathematical" proof of my alleged misconception turns out to be nothing but ill-conceived, the principle of maximum yield still provides the best practice in managing renewable resources.

Based on their fallacy, the linear programming *example* put forth by Chang et al. (2019) is bound to founder. The (arbitrary) illustration pretends to "demonstrate empirically" that it pays off to dismiss the rule of Joseph II in favour of the land expectation model. However, an economically unreasonable calculation cannot serve its purpose.

By the way, I have shown numerically how to establish a profit-maximizing synchronized forest when starting from scratch. Albeit seedlings are initially financed by credit, at the end of the gestation period the entrepreneur is in cash and will receive the highest possible future gain.[14] Knoke et al. (2019) devote the body of their paper to a similar question. As a starting point, however, they do not assume fallow land but an already existing normal forest. The authors then analyse at great length the "… opportunity costs of small changes, e. g. for holding the oldest stand one year longer or changing the age class structure to transition from one fully regulated forest to another with one year longer rotation".[15]

Obviously, this research object has nothing to do with the problem I addressed in my contribution: It was about identifying the profit-maximizing rotation period, nothing more and nothing less. Once this optimal arrangement of timber

[8] Helmedag (2018, p. 302).

[9] Chang et al. (2019, p. 1).

[10] Chang et al. (2019, p. 1).

[11] Knoke et al. (2019, p. 2) also bring in a total area A.

[12] Chang et al. (2019, p. 2).

[13] Chang et al. (2019, p. 1).

[14] See Helmedag (2008, p. 165 f.).

[15] Knoke et al. (2019, p. 2).

production is implemented, there is no need to calculate "… the relevant opportunity costs occurring over the transition period, associated with the activities to convert a fully regulated forest with rotation T to another fully regulated forest with rotation $T+\Delta T$".[16]

Besides, Knoke et al. (2019) distort my approach. Without reference and contrary to the truth, they assert: "For his optimization, Helmedag demanded that the forest net revenues cover the opportunity costs of holding an average forest stand which he termed mid-level opportunity costs".[17] Yet, I did not postulate the *payment* of any opportunity costs, which usually denote the renounced *benefits* of the next best alternative. Of course, revenues, e.g. from selling a firm, can be invested wherever at (identical) interest. But, as mentioned earlier, in equilibrium such activities never entail extra profits. Actually, Knoke et al. (2019) commit the same fundamental error as Chang et al. (2019): again, just like in Eq. (9), the value of total assets is curtailed by the standing timber's present value. In fact, this deduction proves economically flawed because in the model under consideration no such "mid-level opportunity costs" are disbursed.

Finally, Knoke et al. (2019) arrive at "… a rotation shorter than the optimal Faustmann rotation … and much shorter than Helmedag's preferred rotation".[18] However, one should be particularly sceptical about this advice, because the greater the deviation from the principle of maximum sustainable yield, the greater the misallocation of resources.

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[16] Knoke et al. (2019, p. 2 f.).

[17] Knoke et al. (2019, p. 2).

[18] Knoke et al. (2019, p. 5).